$$3S_r + 2S_m = \sigma, \text{ where } \sigma \leqq \sigma_u, \text{ for } 10^6 \text{ cycles life} \tag{74}$$

[Equation (9) in the previous analyses], must be limited by the yield strength, σ_y, for large mean stresses as shown in Figure 62, i.e.

$$2S_{max} = 2S_r + 2S_m \leq \sigma_y \tag{75}$$

A conservative shear-fatigue relation is the following:

$$\left(\frac{3\sigma_y}{\sigma_u}\right) S_r + 2S_m = \sigma_y \quad , \quad \text{for } 10^6 \text{ cycles life} \tag{76}$$

Relation (76) is also shown in Figure 62. [The coefficient $A_n = 3$ in Equations (74) and (76) is taken from data in Reference (35) as indicated earlier on page 164.]

The significance of the limit $S_m = 0$ [used in conjunction with Equation (7) on page 163] is now pointed out. S_m at the bore is related to $(\sigma_\theta)_m$ as follows:

$$S_m = \frac{(\sigma_\theta)_m}{2} + \frac{(p_o - q_o)}{4} = \frac{(\sigma_\theta)_m}{2} + \frac{p_o}{4} \text{ for } q_o = 0 \quad .$$

Thus,

$$(\sigma_\theta)_m = -\frac{p_o}{2} \text{ for } S_m = 0 \quad . \tag{77}$$

For a multiring container it was found that $\left((p_o)_{max} \approx \sigma_u \text{ for } \alpha_r = \frac{(\sigma_\theta)_r}{\sigma_u} = 0.5, \ \alpha_m = \frac{(\sigma_\theta)_m}{\sigma_u} = -0.5 \text{ for } 10^4\text{-}10^5 \text{ cycles life} \right)$. Therefore, the maximum tensile strength fatigue criterion with $\alpha_r = 0.5$, $\alpha_m = -0.5$ is equivalent to $S_m = 0$ for the shear strength criterion.

Coefficients A_n and B_n in Equation (73a) are now calculated for the tensile criterion postulated for high-strength steels ($\sigma_u \geq 250,000$ psi) from the fatigue data given in Table XLII and XLIII. These data are as follows in terms of α_r and α_m:

	Semirange Parameter, α_r	
Fatigue Life, cycles	for $\alpha_m = 0$	for $\alpha_r = \alpha_m$
10^4-10^5	0.50	0.35
10^6-10^7	0.35	0.25

Thus, for $0 \leq \alpha_m \leq \alpha_r$ (zero to a positive mean stress) the coefficients A_n and B_n are calculated to be:

Fatigue Life, cycles	A_n	B_n
10^4-10^5	2.00	0.86
10^6-10^7	2.86	1.14

For, $-\alpha_r \leq \alpha_m \leq 0$, in leiu of actual data, the fatigue relation (73a) is assumed to be horizontal (Figure 61), i.e., $B_n = 0$ with $A_n = 2.00$ and $A_n = 2.86$ for 10^4-10^5 and 10^6-10^7 cycles life, respectively.

General Analysis of Multiring Containers

A multiring container or a multiring unit of a two-unit container such as has been shown in Figure 40, is assumed to have pressures fluctuating between q_o and p_o in the bore and between q_N and p_N on the outside diameter. Minimum stresses during the cycle occur at pressure preloadings q_o and q_N, and maximum stresses occur at operating-pressure loadings of p_o and p_N. (The pressures q_N and p_N are the so called "fluid-support pressures".) The generalized fatigue criteria (73a, b) are used. The elasticity solutions for the stress components in Equations (73a, b) are as follows:

$$(\sigma_\theta)_r = \frac{1}{2(k_n^2 - 1)} \left[(p_{n-1} - q_{n-1})(k_n^2 + 1) - 2(p_n - q_n)k_n^2 \right] , \qquad (78a, b)$$

$$(\sigma_\theta)_m = \frac{1}{2(k_n^2 - 1)} \left[(p_{n-1} + q_{n-1})(k_n^2 + 1) - 2(p_n + q_n)k_n^2 \right] , \qquad (79a, b)$$

$$S_r = \frac{k_n^2}{2(k_n^2 - 1)} \left[(p_{n-1} - p_n) - (q_{n-1} - q_n) \right] .$$

The p_n are related to the q_n as follows:

$$p_n = q_n + (-\sigma_{rn}) , \qquad (80a)$$

where

$$\sigma_{rn} = \frac{(p_O - q_O)}{(K^2 - 1)} (1 - k_{n+1}^2 \; k_{n+2}^2 \; \ldots \; k_N^2) \qquad (80b)$$

$$- \frac{(p_N - q_N)}{(K^2 - 1)} (K^2 - k_{n+1}^2 \; k_{n+2}^2 \; \ldots \; k_N^2) , \; n = 1, 2, \ldots, N-1$$

There are (2N-1) unknowns: N pressures p_n, (n = 0, 1, . . . , N-1) and N-1 pressure q_n, n = 1, 2, . . . , N-1. (Determining p_o the bore pressure determines the pressure capability.) There are also (2N-1) equations: N equations from Equations (73a) or (79b) for rings n = 1, 2, . . . , N and (N-1) equations from Equation (80a). Therefore a solution is tractable.

This analysis was programmed into a computer code, Program MULTIR (abbreviation for multiring), for Battelle's 3400 and 6400 CDC computers. Results are given later when specific designs are discussed. First, the influence of "fluid-support pressures" q_N and p_N is studied by considering the example of a fatigue shear strength design.